A Generalized Bubble Diameter Correlation for Gas-Solid Fluidized Beds

A new bubble diameter correlation is derived to predict bubbling characteristics of fluidized beds of varieties of powders. The present model is founded on the postulate that the steady bubble size, which is often called the maximum stable diameter, observed in a bed of Geldart group A powder is formed as a result of an equilibrium of successive coalescence and splitting. For the cases of group B powders the present correlation automatically converges to the conventional correlation of Mori and Wen (1975), whose predictions are close to those of Rowe (1976) and Darton et al. (1977). For group A powders the present correlation is validated by comparison with experimental data in the literature. Based on this correlation a theoretical explanation is presented for the fact that the maximum bubble diameters observed were up to 50 to 100 times as large as those from the stable bubble theory of Harrison et al. (1961).

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Introduction

Bubble size is the most important index in fluidized-bed reactor design, since it controls most of the rate phenomena in the bed such as bubble rising velocity, gas interchange rate between phases, particle circulation rate, heat transfer, and elutriation of fine particles from the bed surface. It is widely recognized that bubbles are much smaller in beds of fine particles than those in coarse-particle beds, and that above a certain height the bubbles cease to change their average size. The design and operation of fine-particle beds have been important fields of fluidization engineering, since most of the fluidized catalysts are very fine particles. Problems associated with fine particles are drawing even more attention in the field of gas-solid reaction processes due to the recent development of many novel systems, such as circulating fluidized beds.

In the literature a number of bubble diameter correlations have been proposed (Kato and Wen, 1969; Geldart, 1970–1971; Chiba et al., 1973; Mori and Wen, 1975; Hirama et al., 1975; Rowe, 1976; Darton et al., 1977), but the applicability of almost all the proposed correlations is restricted to coarse particles, i.e., group B or D powders according to the Geldart (1973) classification. No correlation with any theoretical background has so far been presented for fine (group A) powders. However, as demonstrated by Miyauchi and Furusaki (1974) and more recently by Werther (1984), accurate simulation of fluidized catalytic reactors is possible as long as the bubble size employed in the model is realistic. Thus we can state that among numerous factors in fluidized-bed catalytic reactor modeling, the prediction of bubble size in beds of fine powders is still a matter of the most urgent necessity for reliable reactor modeling.

To derive a bubble diameter correlation applicable to both group A and B powders we must look back at the controversy on the problem of fluidization modes. From the early period of fluidization study two distinct fluidization modes have been recognized: homogeneous and bubbling, or in other terminology, particulate and aggregative fluidizations. Wilhelm and Kwauk (1948) studied eighteen air- and water-fluidized systems and found bubblelike phenomena in a lead-shot water-fluidized bed. Photographic evidence of such phenomena was presented later by Harrison et al. (1961). Wilhelm and Kwauk suggested the Froude number u_{mf}^2/gd_p as the index to separate two fluidization modes. Harrison et al. (1961) and Davidson and Harrison (1963) postulated that bubbles are filled with particles supplied from their wake and are no longer stable if their rising velocity exceeds the terminal velocity of bed particles. From this postulate they presented the concept of a maximum stable bubble with a theoretical correlation. Since then the question of why, how, and in what situation bubbles can exist in a fluidized bed has been studied by a number of investigators. Experimental data and observations in the literature allow us to list the following as the major factors that determine the fluidization mode and characteristics of bubbling in a given bed:

1. Design parameters such as bed diameter, bed height, distributor type, and effective orifice number on the grid
2. Superficial fluidizing velocity

3. Minimum fluidization velocity
4. Minimum bubbling velocity
5. Gas velocity in the emulsion phase
6. Bubble shape and probable entrainment of particles from the wake
7. Knife or finger formation and bubble splitting
8. Instability of homogeneously fluidized bed

In regard to the bubbling-to-homogeneous phase transition problem the following four aspects of the phenomena have been discussed.

1. Bubble flattening with increased fluid density and with decreased particle size or density (Harrison et al., 1961; Rowe and Partridge, 1965; Kawabata et al., 1981).

2. Increased bubble splitting with decreased particle size (Toei et al., 1974a).

3. Increased interval between minimum fluidization and minimum bubbling with decreased particle size irrespective of particle density (Geldart, 1973; Abrahamsen and Geldart, 1980) or with increased pressure (Varadi and Grace, 1978; Crowther and Whitehead, 1978, this item is closely related to the next.

4. Increased stability of homogeneous suspension with decreased Archimedes number (Davidson and Harrison, 1963; Molerus, 1967; Verloop and Heertijes, 1970; Oltrogge, 1972) or with other mechanisms some of which are related to the splitting (Anderson and Jackson, 1968; Clift and Grace, 1974; Garg and Pritchett, 1975; Didwania and Homsy, 1981).

Correspondingly, the absence of bubbles in a liquid-fluidized bed can be understood as being caused by complete bubble flattening or 100% wake fraction, by infinite bubble splitting, by sufficiently large u_{mb}, or by stabilization of homogeneous suspension, but no unified explanation has so far been established. Here, Davidson's maximum stable bubble concept is not likely to be applicable due to the evidence of bubbles much greater than that (Horio and Wen, 1977), and to the experimental observation that bubble shape does not depend on bubble size. Therefore, if prediction of bubble size, instead of bubbling-to-homogeneous phase transition, is the present problem, bubble splitting is the most important factor in the modeling.

Evidences of bubble splitting were first presented by Rowe et al. (1964) for a two-dimensional bed and by Rowe and Partridge (1965) for a three-dimensional bed. Clift and Grace (1972) showed that bubble breakup takes place not only in gas-solid systems but also in liquid-fluidized beds as well as in bubble columns. On the mechanism of bubble breakup, they found that it is initiated mostly from bubble roof, not from the wake as was expected by Davidson and Harrison (1963), and pointed out that the case should be treated as a Taylor instability problem. Toei et al. (1974a) made a quantitative measurement of bubble splitting in a two-dimensional bed and found that the bubble splitting frequency does not depend on bubble diameter but does depend on the particle size. Their results are shown in Figures 1 and 2. Toei et al. (1974a) then presented a successful model simulation of the growth of disturbances on the bubble roof assuming that the emulsion phase behaves as a perfect fluid. Their findings are important since they are counter evidence to the Davidson and Harrison concept of a maximum stable bubble, where it is assumed that bubble stability depends on bubble size. Therefore, Horio and Wen (1977) concluded that the constant bubble size reported by Morooka et al. (1972) and by Matsen (1973) for beds of fine particles must be explained by a balance of bubble coalescence and splitting. If this is the case, the bubble of constant diameter should not be called a maximum stable bubble but should be called an equilibrium bubble that is always repeating coalescence and splitting.

The objective of the present paper is to establish a new bubble diameter correlation that can cover all noncohesive powders regardless of their size, and to derive an equation for the equilibrium bubble diameter. In constrast to the bubble coalescence problem, which is purely a hydrodynamic problem and has been solved completely (Toei et al., 1967; Clift and Grace 1971), the theory for the bubble splitting problem has been developed rather slowly because it requires much fundamental analysis of the properties of emulsion phase itself. Thus, as far as the authors' knowledge is concerned only the work by Toei et al. (1974a) gives quantitative data for bubble splitting frequency. Werther (1983) presented a bubble diameter formula for fine powders, introducing a critical height above which no bubble growth takes place, but no correlation has yet been proposed for it.

In the following section a new bubble growth model is derived based on the coalescence-splitting mechanism. Visual observation is made to validate this mechanism. Empirical correlations are used for both coalescence and splitting frequencies. The model is then compared with previous data in the literature.

Description of the Model

The present work is founded on the understanding that whether bubble diameter increases or decreases is determined by the balance of coalescence and splitting frequencies. To develop a theory of bubble size distribution quantitative expressions are needed for both the rate of coalescence and the rate of splitting. To provide rational estimates of these parameters the following postulates can be assumed:

1. The coalescence process obeys the dynamics of an ideal fluid and is determined completely by the diameters of interacting bubbles and their mutal location (Clift and Grace, 1971).

2. Previous bubble diameter correlations are valid only for medium to coarse (group B and D) powders, where bubble splitting is negligible. Thus the true frequency of coalescence can be determined directly from the previous bubble diameter correlations.

3. From postulates 1 and 2 the expression for the frequency of bubble coalescence obtained for medium to coarse particles also holds for fine (group A) particles.

4. The frequency of bubble splitting does not depend upon bubble diameter but does depend upon particle diameter. Toei et al. (1974a) data show that f_s^* is proportional to $d_p^{-0.6}$ or possibly to $d_p^{-0.5}$. Tone et al. (1974) reported that bubble diameter decreases with increasing temperature. This suggests that f_s^* is also dependent on fluid viscosity. Therefore in the present work the following expression is assumed for the splitting frequency:

$$f_s^* = \alpha u_{mf}^{-p} \quad (p = 1 \sim 1.2) \tag{1}$$

5. The growth of bubbles is determined by the following effective coalescence frequency:

$$n_{\text{eff}}(\text{effective coalescence frequency}) = n_c(\text{real coalescence frequency}) - n_s(\text{splitting frequency}) \tag{2}$$

where n denotes the frequency per unit height and is related to the bubble frequency f_b by

$$n_{\text{eff}} = n_c - n_s = -\frac{df_b}{dz} \tag{3}$$

Since the bubble frequency f_b in the above equation is related to bubble diameter by

$$f_b = \frac{A_t(u_0 - u_D)}{(\pi/6)D_b^3} \tag{4}$$

$n_c - n_s$ can be related to bubble diameter as in the following:

$$\frac{dD_b}{dz} = \frac{\pi D_b^4}{18(u_0 - u_D)A_t}(n_c - n_s) \tag{5}$$

In Eqs. 4 and 5 a parameter u_D is used instead of u_{mf} to consider the effect of dense phase expansion in beds of fine particles. Approximately, u_D is expected to be identical to the average gas velocity in the dense phase, for which a correlation has been proposed by Abrahamsen and Geldart (1980). For coarse particles, u_D equals u_{mf}, but for fine particles u_D/u_{mf} varies from 1 to 4. However, in the case of fine particles the practical gas velocity is much greater than u_{mf} and not much error is included in the approximation of $u_D = u_{mf}$. In the following bubble diameter calculation $u_D = u_{mf}$ is assumed. If the present correlation is applied to a low-velocity region, more accurate estimation of u_D is required. Also in the reaction model careful assessment of u_D is needed, because predicted conversions may be seriously affected by a slight error in u_D.

Coalescence frequency

From postulate 2 and with the relationship of Eq. 5 we have the following equation for group B or D powder where n_s is negligibly small:

$$\frac{dD_b}{dz} = \frac{\pi D_b^4}{18(u_0 - u_D)A_t} n_c \quad \text{(group B or D powder)}$$

Then rearranging the above expression, we have

$$n_c = \frac{18}{\pi}\frac{(u_0 - u_D)A_t}{D_b^4}\frac{dD_b}{dz} \quad \text{(group B or D powder)} \tag{6}$$

For group B and D powders the following Mori and Wen (1975) correlation has been proved valid for a wide range of operating conditions with an accuracy of ±50%:

$$\frac{D_{bm} - D_b}{D_{bm} - D_{b_0}} = \exp\left(-0.3\frac{z - z_0}{D_t}\right) \tag{7}$$

where D_{b0} and D_{bm} respectively denote the initial bubble diameter and the maximum attainable bubble diameter from coalescence and are given by Mori and Wen as in the following:

$$D_{b0} = \begin{cases} 1.38g^{-0.2}[(u_0 - u_D)A_t/n_{or}]^{0.4} & \text{(8a)} \\ 3.77(u_0 - u_D)^2/g & \text{(8b)} \end{cases}$$

$$D_{bm} = 2.59g^{-0.2}[(u_0 - u_D)A_t]^{0.4} \tag{9}$$

where u_{mf} in the original expressions has been replaced by u_D.

Differentiating Eq. 7 by the height z, we have Eq. 10, and substituting it into Eq. 6 the final expression for n_c is derived as Eq. 11.

$$\frac{dD_b}{dz} = 0.3\frac{D_{bm} - D_b}{D_t} \quad \text{(for group B and D powders)} \tag{10}$$

$$n_c = 1.35(u_0 - u_D)D_t\frac{D_{bm} - D_b}{D_b^4} \quad \text{(for all groups)} \tag{11}$$

where the reason that Eq. 11 is applicable to group A as well as group B and D powders is already stated in postulates 1–3.

Splitting frequency

To determine n_s, the splitting frequency per unit height, the following simple relationship can be applied:

$$n_s = \begin{pmatrix}\text{number of splittings while a} \\ \text{single bubble rises a unit height}\end{pmatrix} \cdot \begin{pmatrix}\text{bubble} \\ \text{frequency}\end{pmatrix} = \frac{f_s^*}{u_b} \cdot f_b \tag{12}$$

Substituting Eq. 4 into Eq. 12, we have

$$n_s = 1.5\frac{(u_0 - u_D)D_t^2}{D_b^3}\frac{f_s^*}{k_b\sqrt{gD_b}} \tag{13}$$

where the bubble velocity coefficient k_b is defined by

$$u_b = k_b\sqrt{gD_b} \tag{14}$$

Equilibrium bubble diameter

In the region sufficiently apart from the distributor level there would be no more change in bubble frequency nor in diameter. Thus from Eq. 3 or 5 we have the following condition for the equilibrium state:

$$n_c = n_s \tag{15}$$

After substituting Eq. 11 and 13 into the above condition and solving it with respect to D_b, we obtain the following equation for equilibrium bubble diameter:

$$\frac{D_{be}}{D_t} = \frac{[-\gamma_M + (\gamma_M^2 + 4D_{bm}/D_t)^{0.5}]^2}{4} \tag{16}$$

where γ_M is defined by

$$\gamma_M = \frac{f_s^*}{0.9k_b}\left(\frac{D_t}{g}\right)^{0.5} \tag{17}$$

Axial bubble diameter distribution

Substituting Eqs. 11 and 13 into Eq. 5 and integrating it under the initial condition $D_b = D_{b0}$ at $z = z_0$, the following general correlation for the axial bubble size distribution is

Table 1. Previous Bubble Diameter Correlations for Group B and D Particles and Corresponding Expressions for Coalescence Frequency and Bubble Diameter

Parameter	Correlation		
	Mori and Wen (1975)	Darton et al. (1977)	Rowe (1976)
n_c	$1.35\dfrac{(u_0-u_{mf})(D_{bm}-D_b)}{D_b^4}D_t$	$1.67\dfrac{(u_0-u_{mf})^{1.5}}{D_b^{4.25}g^{0.25}}D_t^2$	$3.38\dfrac{(u_0-u_{mf})^{1.67}}{D_b^{4.33}g^{0.333}}D_t^2$
dD_b/dz	$0.3(D_{bm}-D_b)/D_t-\gamma D_b^{0.5}$	$\gamma_D D_b^{-0.25}-\gamma D_b^{0.5}$	$\gamma_R D_b^{-1/3}-\gamma D_b^{0.5}$
D_b	$\left(\dfrac{\sqrt{D_b}-\sqrt{D_{be}}}{\sqrt{D_{b0}}-\sqrt{D_{be}}}\right)^{1-(\gamma_M/\eta)}\left(\dfrac{\sqrt{D_b}-\sqrt{\delta}}{\sqrt{D_{b0}}-\sqrt{\delta}}\right)^{1+(\gamma_M/\eta)}=\exp\left(-0.3\dfrac{z-z_0}{D_t}\right)$	$\ln\left(\dfrac{\beta_1}{\beta_{10}}\right)-\dfrac{4}{\sqrt{3}}(\tan^{-1}\beta_2-\tan^{-1}\beta_{20})-2\beta^2\beta_3=\beta^3\gamma^{2/3}(z-z_0)$	Analytical expression not yet available
D_{be}	$(D_t/4)(-\gamma_M+\eta)^2$	$(\gamma_D/\gamma)^{4/3}$	$(\gamma_R/\gamma)^{1.2}$

$\gamma=f_s^*/3k_b\sqrt{g}$, $\gamma_M=\gamma\sqrt{D_t}/0.3$, $\gamma_D=0.37(u_0-u_{mf})^{0.5}/g^{0.25}$, $\gamma_R=0.75(u_0-u_{mf})^{2/3}/g^{1/3}$
$\delta=(\gamma_M+\eta)^2D_t/4$, $\eta=(\gamma_M^2+4D_{bm}/D_t)^{0.5}$
$\beta=(\gamma_D/\gamma)^{-1/3}$, $\beta_1=1-3/(\beta^{1/2}D_b^{1/8}-\beta^{-1/2}D_b^{-1/8})^2$, $\beta_2=(1+2\beta D_b^{0.25})/\sqrt{3}$
$\beta_3=D_b^{0.5}-D_{b0}^{0.5}$, $\beta_{10}=\beta_1(D_b-D_{b0})$, $\beta_{20}=\beta_2(D_b-D_{b0})$

obtained:

$$\left(\frac{\sqrt{D_b}-\sqrt{D_{be}}}{\sqrt{D_{b0}}-\sqrt{D_{be}}}\right)^{1-\gamma_M/\eta}\left(\frac{\sqrt{D_b}+\sqrt{\delta}}{\sqrt{D_{b0}}+\sqrt{\delta}}\right)^{1+\gamma_M/\eta}=\exp\left(-0.3\frac{z-z_0}{D_t}\right) \quad (18)$$

where parameters δ and η are defined as follows:

$$\delta/D_M=(\gamma_M+\eta)^2/4 \quad (19)$$

$$\eta=(\gamma_M^2+4D_{bm}/D_t)^{0.5} \quad (20)$$

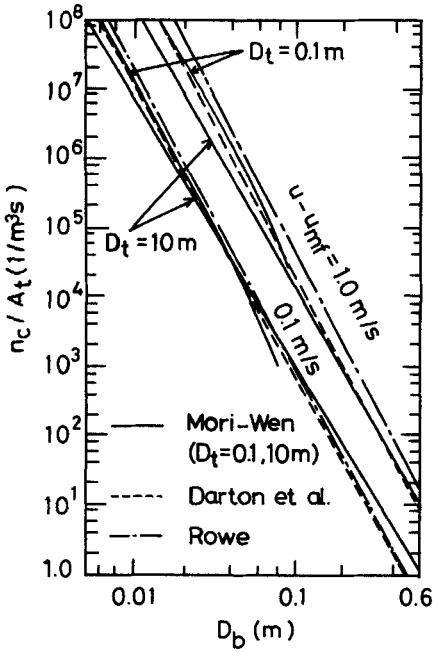

Figure 1. Coalescence frequencies from previous correlations.

In the case of bubble growth with negligible splitting, i.e., $f_s \to 0$, these parameters approach the following values:

$$\gamma_M \to 0,\ D_{be} \to D_{bm},\ \delta \to D_{bm},\ \eta \to 2\sqrt{D_{bm}/D_t} \quad (21)$$

Therefore, Eq. 18 converges to the Mori and Wen correlation, Eq. 7, for the case of group B or D powders.

Similar derivation from other correlations

In addition to the Mori and Wen correlation, two other correlations recently proposed are examined: the Darton et al. (1977) and Rowe (1976) correlations. The mathematical procedure is the same as for the case of the Mori and Wen correlation. The results are listed in Table 1. Coalescence frequencies from these correlations are compared in Figure 1. As can be seen from the figure all three correlations give almost equal coalescence frequency. However, the final expression for bubble diameter from the Darton et al. correlation becomes much more complicated than that from the Mori and Wen correlation, Equation 18. For the case of the Rowe correlation analytical integration of Eq. 5

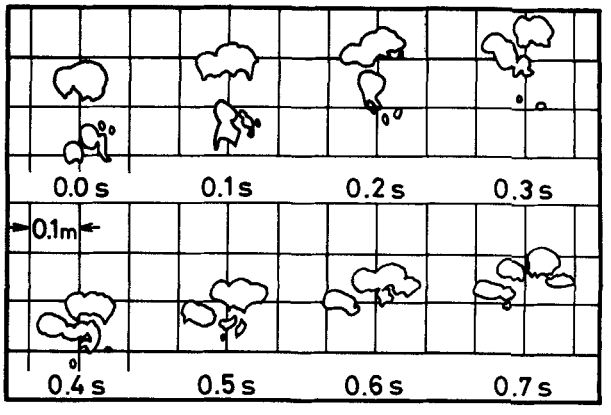

Figure 2. Behavior of two-dimensional bubbles in an equilibrium of coalescence and splitting.
FCC particles, ρ_b = 460 kg/m³, u_0 = 0.062 m/s

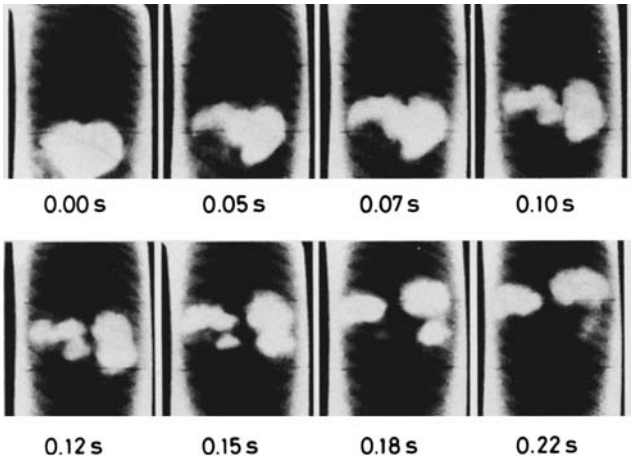

Figure 3. Behavior of three-dimensional bubbles after injection into incipiently fluidized FCC bed.

seems difficult. Therefore, although the results from the above three correlations would not differ much, Eq. 18 is recommended for practical use because of its simplicity.

Experimental Observation of Equilibrium Bubbles

Fresh FCC catalyst particles (d_{psv} = 57 μm) were fluidized by nitrogen in a two-dimensional bed of 0.5 m inside width, 1.76 m height from the distributor to the top, and 12 mm thickness, as well as in a three-dimensional bed of 0.099 m ID. Entrained particles were recycled from the cyclone to the bed. The distributor of the two-dimensional bed was a 10 mm thick PMMA plate having fifteen 2 mm orifices in 33.3 mm pitch. The lower surface of the distributor was covered by stainless steel gauze to prevent particle falling. A sintered stainless steel plate was used for the three-dimensional bed distributor. Bubble behavior was observed visually (by X-ray in the case of the three-dimensional bed) and recorded by a video recorder (SONY U-MATIC VO-5800 with a 1/500 s rotary shutter camera, SONY RSC-1110) at 60 frames/s.

Typical behavior of rising bubbles in the two-dimensional bed under a freely bubbling condition is shown in Figure 2, where it is obvious that the time-averaged bubble size remains roughly the same as a result of continuous coalescence and splitting. Figure 3 shows the behavior of a single bubble injected into an incipiently fluidized bed from a nozzle located 0.11 m above the distributor. Bubble splitting took place as often in the three-dimensional bed as in the two-dimensional bed. Although it is qualitative, it has been confirmed from these observations that the splitting-coalescence mechanism dominates the process of bubble size change regardless of the dimension of a bed and that there is no stable maximum bubble in a bed of group A powder.

Validation of the Theory

Methodology

In order to calculate bubble diameters from Eqs. 16 and 18, it is necessary to know the bubble splitting frequency f_s^*. As stated in postulate 4, Eq. 1 is assumed in the present work.

Substituting Eq. 1 into Eq. 15, γ_M becomes:

$$\gamma_M = \frac{\alpha u_{mf}^{-p}}{0.9 k_b}\left(\frac{D_t}{g}\right)^{0.5} \tag{22}$$

From the data of Toei et al. (1974a, b) shown in Figure 1 the value of α can be determined as 0.065 m/s^2 for p = 1 or 0.036 m$^{1.2}$/s$^{2.2}$ for p = 1.2. However, these values are for two-dimensional bubble splitting and would not be valid for three-dimensional bubbles. Quite a lot of work is needed to accumulate sufficient bubble splitting data in three-dimensional beds with an X-ray method. Therefore, in the present paper the quotient α/k_b is determined backward from the experimental data of the average bubble size so that the best prediction is obtained. In this context, the theory is understood to be validated if the adjusted parameter α/k_b turns out to be a constant regardless of the types of powders and of the scale of the fluidized beds. In addition it is also required that the bubble diameter distributions calculated from the parameter α/k_b determined above agree with the observed distributions over the whole range of the bed height.

Determination of α/k_b

Adjuted values of α/k_b are shown in Figure 4. For wide ranges of u_0 (= 0.05 ~ 0.5 m/s) and D_t (= 0.079 ~ 1.0 m) the parameter α/k_b only changes within ±20 around the following mean values regardless of the value of parameter p:

$$(\alpha/k_b) = \begin{cases} 2.3 \times 10^{-2}\ \text{m/s}^2 & (\text{for } p = 1) \quad (23a) \\ 6.5 \times 10^{-3}\ \text{m}^{1.2}/\text{s}^{2.2} & (\text{for } p = 1.2) \quad (23b) \end{cases}$$

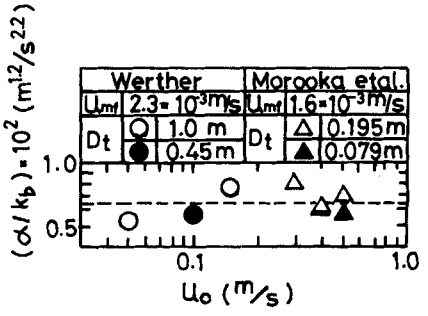

Figure 4. Parameter α/k_b determined from experimental data of D_{be}.

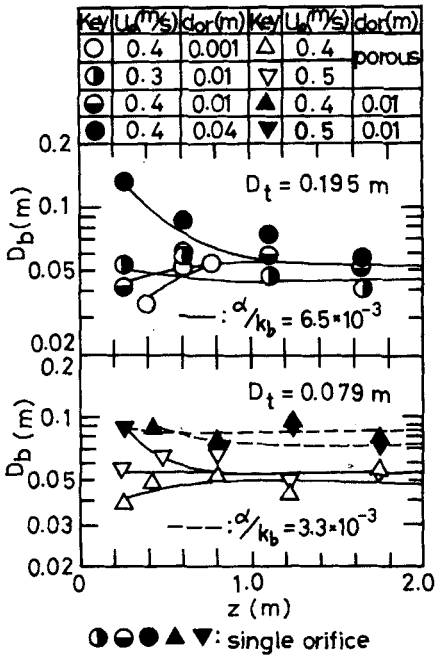

Figure 5. Comparison of predicted bubble size distribution with data of Morooka et al. (1971).
FCC particles, u_{mf} = 1.6 mm/s

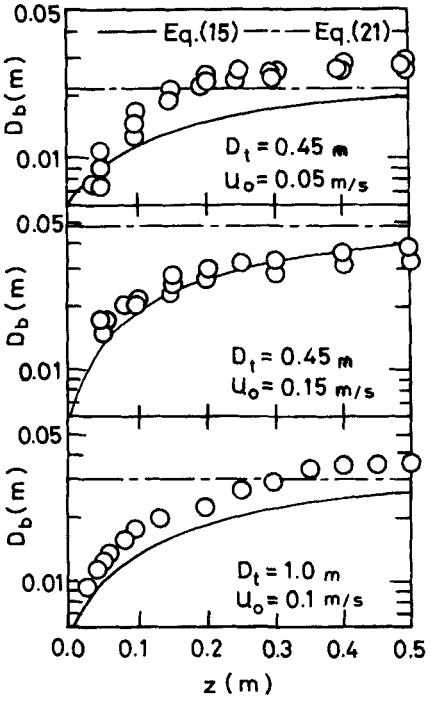

Figure 6. Comparison of predicted bubble size distribution with data of Werther (1984).
FCC particles, u_{mf} = 2.3 mm/s

Corresondingly, the final expression for γ_M is given as follows:

$$\gamma_M = \begin{cases} 2.56 \times 10^{-2}(D_t/g)^{0.5}/u_{mf} & (\text{for } p = 1) \quad (24a) \\ 7.22 \times 10^{-3}(D_t/g)^{0.5}/u_{mf}^{1.2} & (\text{for } p = 1.2) \quad (24b) \end{cases}$$

If $k_b = 0.711$ (Davidson and Harrison, 1963) is used, the values of α in three-dimensional beds obtained from Eq. 23 are 0.016 m/s^2 for $p = 1$ and 0.0046 $m^{1.2}/s^{2.2}$ for $p = 1.2$. These values are smaller than those from the Toei et al. (1974) data for a two-dimensional bed but are of the same order of magnitude. To establish a more general correlation including the effect of particle and gas properties on α/k_b, more fundamental work on bubble splitting is definitely needed. Therefore, for the time being the above equations for the case of $p = 1$, Eqs. 23b and 24a, are recommended for practical application of the present bubble correlation.

Axial bubble diameter distribution

Comparisons of Eq. 18 with the experimental data of Morooka et al. (1971) and Werther (1984) are made in Figures 5 and 6, and good agreement between theory and observation has been obtained. In Figure 5 the bubble size reduction observed when initial bubbles are larger than equilibrium bubbles is well reproduced by the present model. In Figure 7 a comparison of the present correlation with previous ones is made for one case of Werther data. It can be seen that all of the previous correlations much overestimate bubble diameter due to the lack of splitting mechanism.

The bubble diameter correlation, which can cover group A to D powders, is thus satisfactorily validated over a wide range of operating conditions. This correlation allows us to develop more accurate and universal fluidized-bed reactor models than those previously proposed. In addition, this correlation may help to terminate the controversy on equilibrium bubble size that was first raised by Harrison et al. (1961) about two decades ago. The equilibrium bubble diameter from the present theory is compared with the maximum stable bubble diameter from Davidson and Harrison theory in Figure 8. It is found that the equilibrium bubble diameter for fine powders is 50 to 100 times larger than Davidson's maximum stable diameter. This result also agrees with observations reported from a large-scale fluidized bed (Matsen, 1973).

Conclusions

A general bubble diameter correlation that takes into account both splitting and coalescence for varieties of powders including

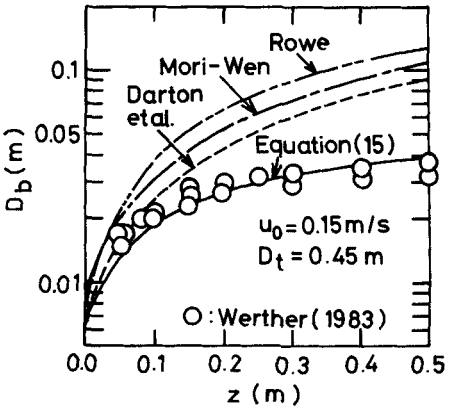

Figure 7. Comparison of proposed and previous correlations.

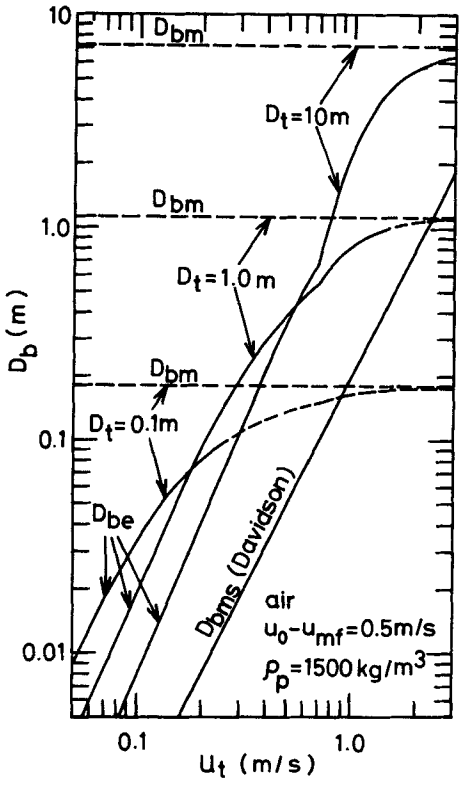

Figure 8. Equilibrium bubble diameter compared with maximum stable bubble diameter of Harrison et al. (1961).

Geldart group A powder has been proposed. The correlation is expressed in an implicit but simple form given by Eq. 18. In the case of coarse particles where splitting is negligible, this correlation converges to the Mori and Wen (1975) correlation. As far as numerical values are concerned, it also converges to the Darton et al. (1977) and Rowe (1976) correlations.

The equilibrium bubble diameter D_{be} is also correlated in a simple form given by Eq. 16.

By comparing predicted bubble diameters with those observed by Morooka et al. (1971) and Werther (1983) the proposed model and correlations have been successfully validated over a wide range of operating conditions ($0.05 \leqq u_0 \leqq 0.5$ m/s, $0.079 \leqq D_t \leqq 1.0$ m).

Model predictions for equilibrium bubble size were compared with the Harrison et al. (1961) maximum stable bubble diameter. Previous observations of bubbles up to about 50 times greater than the maximum stable bubble diameter have been theoritically confirmed to be reasonable.

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Notation

A_t = bed cross-sectional area, m^2
$d_p = d_{psv}$, m
d_{psv} = surface volume mean particle diameter, m
D_b = bubble diameter, m
D_{b0} = initial bubble diameter, m
D_{be} = equilibrium bubble diameter, m
D_{bm} = maximum bubble diameter from total coalescence of bubbles, m
D_{bms} = Davidson's maximum stable bubble diameter, m
D_t = column diameter, m
f_b = bubble frequency per total bed cross section, 1/s
f_s^* = splitting frequency of a single bubble, 1/s
g = gravity acceleration, m/s^2
$k_b = u_b/\sqrt{gD_b}$
n_c = real coalescence frequency per unit bed height, 1/m · s
n_{eff} = effective coalescence frequency, 1/m · s
n_s = splitting frequency per unit bed height, 1/m · s
p = power in Eq. 1
u_0 = superficial gas velocity, m/s
u_b = bubble rising velocity, m/s
u_D = average gas velocity in dense phase (superficial velocity), m/s
u_{mf} = minimum fluidization velocity, m/s
u_t = terminal velocity, m/s
z = height above distributor, m
z_0 = height of initial bubble formation, m

Greek letters

α = coefficient in bubble splitting correlation Eq. 1, m^p/s^{1+p}
β = parameter, Table 1, $m^{-0.25}$
β_1, β_2 = parameters, Table 1
β_3 = parameter, Table 1, $m^{0.5}$
γ = parameter, Table 1, $m^{0.5}$
γ_D = parameter from Darton et al. correlation, Table 1, $m^{0.25}$
γ_M = parameter from Mori and Wen correlation, Eq. 17
γ_R = parameter from Rowe correlation, Table 1, $m^{1/3}$
δ = parameter, Eq. 19, m
ϵ_b = bubble fraction
ϵ_D = void fraction of dense phase
ϵ_{mf} = void fraction of bed at minimum fluidization
η = parameter, Eq. 20
μ = viscosity, Pa · s
ρ_b = bulk density, kg/m^3
ρ_g = gas density, kg/m^3
ρ_p = particle density, kg/m^3

Literature Cited

Abrahamsen, A. R., and D. Geldart, "Behavior of Gas-Fluidized Beds of Fine Powders. II: Voidage of the Dense Phase in Bubbling Beds," *Powder Technol.*, **26**, 47 (1980).

Anderson, T. B., and R. Jackson, "Fluid Mechanical Description of Fluidized Bed: Stability of the State of Uniform Fluidization," *Ind. Eng. Chem. Fundam.*, **7**, 12 (1968).

Chiba, T., K. Terashima, and H. Kobayashi, "Bubble Growth in Gas-Fluidized Beds," *J. Chem. Eng. Japan*, **6**, 78 (1973).

Clift, R., and J. R. Grace, "Coalescence of Bubbles in Fluidized Beds," *AIChE Symp. Ser.*, **67**(no.116), 23 (1971).

———, "The Mechanism of Bubble Breakup in Fluidized Beds," *Chem. Eng. Sci.*, **27**, 2309 (1972).

Clift, R., J. R. Grace, and M.E. Weber, "Stability of Bubbles in Fluidized Beds," *Ind. Eng. Chem. Fundam.*, **13**, 45 (1974).

Crowther, M. E., and J. C. Whitehead, "Fluidization of Fine Particles at Elevated Pressure," *Fluidization*, J. F. Davidson, D. L. Keairns, eds., Cambridge, 65 (1978).

Darton, R. C., R. D. LaNauze, J. F. Davidson, and D. Harrison, "Bubble Growth Due to Coalescence in Fluidized Beds," *Trans. Inst. Chem. Eng.*, **55**, 274 (1977).

Davidson, J. F., and D. Harrison, *Fluidized Particles*, Cambridge, London (1963).

Didwania, A. K., and G. M. Homsy, "Rayleigh-Taylor Instabilities in Fluidized Beds," *Ind. Eng. Chem. Fundam.*, **20**, 318 (1981).

Garg, S. K., and J. W. Pritchett, "Dynamics of Gas-Fluidized Beds," *J. Appl. Phys.*, **46**, 4493 (1975).

Geldart, D., "The Size and Frequency of Bubbles in Two- and Three-Dimensional Gas-Fluidized Beds," *Powder Technol.*, **4**, 41 (1970–1971).

———, "Types of Gas Fluidization," *Powder Technol.*, **7**, 285 (1973).

Harrison, D., J. F. Davidson, and J. W. de Kock, "On the Nature of Aggregative and Particulate Fluidization," *Trans. Inst. Chem. Eng.*, **39**, 202 (1961).
Hirama, T., M. Ishida, and T. Shirai, "The Lateral Dispersion of Solid Particles in Fluidized Beds," *Kagaku Kogaku Ronbunshu*, **1**, 272 (1975).
Horio, M., and C. Y. Wen, "An Assesment of Fluidized-Bed Modeling," *AIChE Symp. Ser.*, **73**(no.161), 9 (1977).
Kawabata, J., M. Yamashita, Y. Tazaki, S. Honma, T. Chiba, T. Sumiya, and K. Endo, "Characteristics of Gas-Fluidized Beds Under Pressure," *J. Chem. Eng. Japan*, **14**, 85 (1981).
Matsen, J. M., "Evidence of Maximum Stable Bubble Size in Fluidized Bed," *AIChE Symp. Ser.*, **69**(no.128), 30 (1973).
Miyauchi, T., and S. Furusaki, "Relative Contribution of Variables Affecting the Reaction in Fluid Bed Contactors," *AIChE J.*, **20**, 1087 (1974).
Molerus, O., "The Hydrodynamic Stability of the Fluidized Bed," *Proc. Intl. Symp. Fluidization*, Eindhoven, Netherlands Univ. Press, 134 (1967).
Mori, S., and C. Y. Wen, "Estimation of Bubble Diameter in Gaseous Fluidized Beds," *AIChE J.*, **21**, 109 (1975).
Morooka, S., K. Tajima, and T. Miyauchi, "Behavior of Gas Bubbles in Fluid Beds," *Kagaku Kogaku*, **35**, 680 (1971); *Int. Chem. Eng.*, **12**, 168 (1972.)
Oltrogge, R. D., "Correlation of the Bubble Point and Dense Phase Expansions for Small-Particle Gas-Fluidized Beds," Ph.D. Thesis, Univ. Michigan (1972).
Rowe, P. N., "Prediction of Bubble Size in a Gas-Fluidized Bed," *Trans. Inst. Chem. Eng.*, **31**, 285 (1976).
Rowe, P. N., and B. A. Partridge, "An X-ray Study of Bubbles in Fluidized Bed," *Chem. Eng. Sci.*, **43**, 157 (1965).
Rowe, P. N., B. A. Partridge, and E. Lyall, "Cloud Formation Around Bubbles in Gas Fluidized Beds," *Chem. Eng. Sci.*, **19**, 973 (1964).
Toei, R., R. Matsuno, T. Sumitani, and M. Mori, "The Coalescence of Bubbles in a Gas-Solid Fluidized Bed," *Kagaku Kogaku*, **31**, 861 (1967).
Toei, R., R. Matsuno, M. Oichi, and K. Yamamoto, "Deformations and Splittings of a Bubble in a Two-Dimensional Fluidized Bed: Experimental Results," *J. Chem. Eng. Japan*, **7**, 447 (1974a).
———, "Deformations and Splittings of a Bubble in a Two-Dimensional Fluidized Bed: Theoretical Calculations," *J. Chem. Eng. Japan*, **7**, 451 (1974b).
Tone, S., H. Seko, H. Maruyama, and T. Otake, "Catalytic Cracking of Methylcyclohexane over Silica Alumina Catalyst in Gas-Fluidized Bed," *J. Chem. Eng. Japan*, **7**, 44 (1974).
Varadi, T., and J. R. Grace, "High-Pressure Fluidization in a Two-Dimensional Bed," *Fluidization*, J. F. Davidson, D. L. Keairns, eds., Cambridge, 55 (1978).
Verloop, J., and P. M. Heertijes, "Shock Waves as a Criterion for the Transition from Homogeneous to Heterogeneous Fluidization," *Chem. Eng. Sci.*, **25**, 825 (1970).
Werther, J., "Hydrodynamics and Mass Transfer Between the Bubble and Emulsion Phases in Fluidized Beds of Sand and Cracking Catalyst," *Fluidization*, D. Kunii, R. Toei, eds., Eng. Found., 93 (1984).
Wilhelm, R. H., and M. Kwauk, "Fluidization of Solid Particles," *Chem. Eng. Prog.*, **44**, 201 (1948).

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